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The Reduction of Resolution of Weyl module from Characteristic-free to Lascoux Resolution in case (6,5,3)

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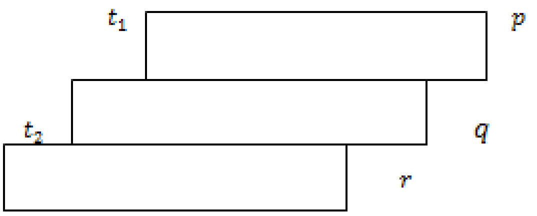
***Abstract*— In this paper we study the relation between the resolution of weyl module $K_{(6,5,3)}F$ in characteristic-free mode and in the Lascoux mode (characteristic zero),more precisely we obtain the Lascoux resolution of $K_{(6,5,3)}F$ in characteristic zero as an application of the resolution of $K_{(6,5,3)}F$ in characteristic-free.**

***Index Terms*— Resolution ,Weyl module Lascoux module ,divided power ,characteristic-free.**

I. Introduction

Let R be commutative ring with 1 and F be free R-module by D_nF we mean the divided power of degree n. The resolution Res [p ,q ,r,t_1, t_2] of weyl module $K_{\lambda/\mu}F$ associated to the three-rowed skew-shape ($p+t_1+t_2, q+t_2, r)/(t_1+t_2, t_2, 0)$ call the shape represented by the diagram



In general ,the weyl module $K_{\lambda/\mu}F$ is presented by the box map

$$\sum_{k>0} D_{p+t_1+k}F\otimes D_{q-t_1-k}F\otimes D_rF$$
$$\oplus$$
$$\xrightarrow{\square} D_pF\otimes D_qF\otimes D_rF \xrightarrow{d'_{\lambda/\mu}} K_{\lambda/\mu}$$
$$\sum_{l>0} D_pF\otimes D_{q+t_2+l}F\otimes D_{r-t_2-l}F$$

Where the maps

$\sum_{k>0} D_{p+t_1+k}F\otimes D_{q-t_1-k}F\otimes D_rF \longrightarrow D_pF\otimes D_qF\otimes D_rF$ may be interpreted as K^{th} divided power of the place polarization from place 1 to place 2 (i.e. $\partial_{32}^{(k)}$) ,the maps

$\sum_{l>0} D_pF\otimes D_{q+t_2+l}F\otimes D_{r-t_2-l}F \longrightarrow D_pF\otimes D_qF\otimes D_rF$ may be place 2 interpreted as l^{th} divided power of the place polarization from place 2 to 3 (i.e. $\partial_{32}^{(l)}$)[1]. we have to mention that we shall use D_n instead of D_nF to refer to divided power algebra of degree n .

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II. Characteristic-Free Resolution of the Partition (6,5,3)

We find the terms of the resolution of weyl module in the case of the partition (6,5,3) .In general a terms of the resolution of weyl module in the case of a three-rowed partition (p,q,r) which appeared in [2] are

Res ([p,q;0]) $\otimes D_r \oplus \sum_{l\geq 0} \underline{Z}_{32}^{(l+1)} y$ Res ([p,q+l+1;l+1]) $\otimes D_{r-l-1} \oplus \sum_{l_1\geq 0, l_2\geq l_1} \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(l_1+1)} z$ Res ([$p+l_1$+1,$q+l_2$+1,l_2-l_1])$\otimes D_{r-(l_1+l_2+2)}$

Where x, y and z stand for the separator variables, and the boundary map is $\partial_x+\partial_y+\partial_z$. Let again Bar(M,A;S) be the free bar module on the set S={x,y,z} consisting of three separators x, y and z, where A is the free associative (non-commutative) algebra generated by Z_{21} ,Z_{32} and Z_{31} and their divided powers with the following relations:

$Z_{32}^{(a)}Z_{31}^{(b)} = Z_{31}^{(b)}Z_{32}^{(a)}$ and $Z_{21}^{(a)}Z_{31}^{(b)} = Z_{31}^{(b)}Z_{21}^{(a)}$

and the module M is the direct sum of tensor products of divided power module $D_p \otimes D_q \otimes D_r$ for suitable p,q and r with the action of Z_{21},Z_{32} and Z_{31} and their divided powers . we will consider the case when $p=6, q=5, and\ r=3$. we have

Res([6,5,0]) $\otimes D_3 \oplus \sum_{l\geq 0} \underline{Z}_{32}^{(l+1)} y$

Res([6,5+l+1;l+1]) $\otimes D_{3-l-1} \oplus \sum_{l_1\geq 0, l_2\geq l_1} \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(l_1+1)} z$

Res([6+l_1+1,5+l_2+1,l_2-l_1])$\otimes D_{3-(l_1+l_2+2)}$,So

$\sum_{l\geq 0} \underline{Z}_{32}^{(l+1)} y$ Res([6,5+l+1;l+1]) $\otimes D_{3-l-1}$

= $\underline{Z}_{32}y$ Res([6,6;1]) $\otimes D_2 \oplus \underline{Z}_{32}^{(2)} y$ Res([6,7;2]) $\otimes D_1 \oplus \underline{Z}_{32}^{(3)} y$ Res([6,8;3])$\otimes D_0$ and

$\sum_{l_1\geq 0, l_2\geq l_1} \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(l_1+1)} z$ Res([6+ l_1 +1,5+ l_2 +1; l_2-l_1]) $\otimes D_{3-(l_1+l_2+2)}$

= $\underline{Z}_{32}y\ \underline{Z}_{31}z$ Res([7,6;0])$\otimes D_1 \oplus \underline{Z}_{32}^{(2)} y\ \underline{Z}_{31}z$ Res([7,7;1])D_0

Where $\underline{Z}_{32}y$ is the bar complex: $0 \longrightarrow Z_{32}y \xrightarrow{\partial_y} Z_{32} \longrightarrow 0$

$\underline{Z}_{32}^{(2)} y$ is the bar complex:

$0 \longrightarrow Z_{32}yZ_{32}y \xrightarrow{\partial_y} Z_{32}^{(2)}y \xrightarrow{\partial_y} Z_{32}^{(2)} \longrightarrow 0$

$\underline{Z}_{32}^{(3)} y$ is the bar complex:

$0 \longrightarrow Z_{32}yZ_{32}yZ_{32}y \xrightarrow{\partial_y} Z_{32}^{(2)}yZ_{32}y \oplus Z_{32}yZ_{32}^{(2)}y$

$\xrightarrow{\partial_y} Z_{32}^{(3)}y \xrightarrow{\partial_y} Z_{32}^{(3)} \longrightarrow 0$ and $\underline{Z}_{31}z$ is the bar complex: $0 \longrightarrow Z_{31}z \xrightarrow{\partial_z} Z_{31} \longrightarrow 0$

Then in this case we have the following terms :

◦ In dimension zero (M_0) we have $D_6 \otimes D_5 \otimes D_3$

◦ In dimension one (M_1) we have

• $Z_{21}^{(b)}xD_{6+b} \otimes D_{5-b} \otimes D_3$; $with\ b = 1,2,3,4,5$

• $Z_{32}^{(b)}yD_6 \otimes D_{5+b} \otimes D_{3-b}$; $with\ b = 1,2,3$

◦ In dimension two (M_2) we have the sum of the following terms:

• $Z_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{5-|b|} \otimes D_3$; $with\ |b| = b_1 + b_2 =$
$2,3,4,5$

• $Z_{32}yZ_{21}^{(b)}xD_{6+b} \otimes D_{6-b} \otimes D_2$; $with\ b = 2,3,4,5,6$

• $Z_{32}^{(2)}yZ_{21}^{(b)}xD_{6+b} \otimes D_{7-b} \otimes D_1$; $with\ b = 3,4,5,6,7$

• $Z_{32}^{(3)}yZ_{21}^{(b)}xD_{6+b} \otimes D_{8-b} \otimes D_0$; $with\ b = 4,5,6,7,8$

• $Z_{32}^{(b_1)}y\, Z_{32}^{(b_2)}yD_6 \otimes D_{5+|b|} \otimes D_{3-|b|}$; $with\ |b| = b_1 + b_2 =$
$2,3$

• $Z_{32}^{(b)}yZ_{31}zD_7 \otimes D_{5+b} \otimes D_{2-b}$; $with\ b = 1,2$

◦ In dimension three (M_3) we have the sum of the following terms:

• $Z_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{5-|b|} \otimes D_3$; $with\ |b| = b_1 +$
$b_2 + b_3 = 3,4,5\ and\ b_1 \geq 1$

• $Z_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{6-|b|} \otimes D_2$; $with\ |b| = b_1 +$
$b_2 = 3,4,5,6$

and $b_1 \geq 2$

• $Z_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{7-|b|} \otimes D_1$; $with\ |b| = b_1 +$
$b_2 = 4,5,6,7$

$and\ b_1 \geq 3$

• $Z_{32}yZ_{32}yZ_{21}^{(b)}xD_{6+b} \otimes D_{7-b} \otimes D_1$; $with\ b = 3,4,5,6,7$

• $Z_{32}^{(3)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|} \otimes D_{7-|b|} \otimes D_1$; $with\ |b| = b_1 +$
$b_2 = 5,6,7,8$

$and\ b_1 \geq 4$

• $Z_{32}^{(c_1)}yZ_{32}^{(c_2)}yZ_{21}^{(b)}xD_{6+b} \otimes D_{8-b} \otimes D_0$; $with\ c_1 + c_2 =$
$3\ and\ b = 4,5,6,7,8$

• $Z_{32}yZ_{32}yZ_{32}yD_6 \otimes D_8 \otimes D_0$

• $Z_{32}yZ_{31}zZ_{21}^{(b)}xD_{7-b} \otimes D_{6-b} \otimes D_1$; $with\ b = 1,2,3,4,5,6$ • $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b)}xD_{7+b} \otimes D_{7-b} \otimes D_0$; $with\ b = 2,3,4,5,6,7$

• $Z_{32}yZ_{32}yZ_{31}zD_7 \otimes D_7 \otimes D_0$

◦ In dimension four (M_4) we have the sum of the following terms:

• $Z_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|} \otimes D_{5-|b|} \otimes D_3$; $with\ |b| =$
$\sum_{i=1}^{4} b_i = 4,5\ and\ b_1 \geq 1$

• $Z_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|} \otimes D_{6-|b|} \otimes D_2$; $with\ |b| =$
$b_1 + b_2 + b_3 = 4,5,6$

and $b_1 \geq 2$

- $Z_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|}\otimes D_{7-|b|}\otimes D_1$; $with\ |b| = b_1+b_2+b_3 = 5,6,7$

$and\ b_1 \geq 3$

- $Z_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|}\otimes D_{7-|b|}\otimes D_1$; $with\ |b| = b_1+b_2 = 4,5,6,7\ and\ b_1 \geq 3$

- $Z_{32}^{(3)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$; $with\ |b| = b_1+b_2+b_3 = 6,7,8$

$and\ b_1 \geq 4$

- $Z_{32}^{(c_1)}yZ_{32}^{(c_2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$; $with\ c_1+c_2 = 3\ and\ |b| = b_1+b_2 = 5,6,7,8$

$and\ b_1 \geq 4$

- $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b)}xD_{6+b}\otimes D_{8-b}\otimes D_0$; $with\ b = 4,5,6,7,8$

- $Z_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{7+|b|}\otimes D_{6-|b|}\otimes D_1$; $with\ |b| = b_1+b_2 = 2,3,4,5,6\ and\ b_1 \geq 1$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$; $with\ |b| = b_1+b_2 = 3,4,5,6,7\ and\ b_1 \geq 2$

- $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b)}xD_{7+b}\otimes D_{7-b}\otimes D_0$; $with\ b = 2,3,4,5,6,7$

◦ In dimension five (M_5) we have the sum of the following terms:

- $Z_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{11}\otimes D_0\otimes D_3$

- $Z_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|}\otimes D_{6-|b|}\otimes D_2$; $with\ |b| = \sum_{i=1}^{4} b_i = 5,6\ and$

$b_1 \geq 2$

- $Z_{32}^{(2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|}\otimes D_{7-|b|}\otimes D_1$; $with\ |b| = \sum_{i=1}^{4} b_i = 6,7\ and$

$b_1 \geq 3$

- $Z_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|}\otimes D_{7-|b|}\otimes D_1$; $with\ |b| = b_1+b_2+b_3 = 5,6,7\ and\ b_1 \geq 3$

- $Z_{32}^{(3)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$; $with\ |b| = \sum_{i=1}^{4} b_i = 7,8$

$and\ b_1 \geq 4$

- $Z_{32}^{(c_1)}yZ_{32}^{(c_2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$; $with\ c_1+c_2 = 3$

$and\ |b| = b_1+b_2+b_3 = 6,7,8\ and\ b_1 \geq 4$

- $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$; $with\ |b| = b_1+b_2 = 5,6,7,8\ and\ b_1 \geq 4$

- $Z_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{7+|b|}\otimes D_{6-|b|}\otimes D_1$; $with\ |b| = b_1+b_2+b_3 = 3,4,5,6\ and\ b_1 \geq 1$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$; $with\ |b| = b_1+b_2+b_3 = 4,5,6,7\ and\ b_1 \geq 2$

- $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$; $with\ |b| = b_1+b_2 = 3,4,5,6,7\ and\ b_1 \geq 2$

◦ In dimension six (M_6) we have the sum of the following terms:

- $Z_{32}yZ_{21}^{(2)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{12}\otimes D_0\otimes D_2$

• $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{13}\otimes D_0\otimes D_1$ • $Z_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|}\otimes D_{7-|b|}\otimes D_1$;

$with\ |b| = \sum_{i=1}^{4} b_i = 6{,}7 and\ b_1 \geq 3$

• $Z_{32}^{(3)}yZ_{21}^{(4)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{14}\otimes D_0\otimes D_0$ • $Z_{32}^{(c_1)}yZ_{32}^{(c_2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$;

$with\ c_1 + c_2 = 3and\,|b| = \sum_{i=1}^{4} b_i = 7{,}8\ \ and\ b_1 \geq 4$ • $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$;

$with\ \ |b| = b_1 + b_2 + b_3 = 6{,}7{,}8\ and\ b_1 \geq 4$ • $Z_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{7+|b|}\otimes D_{6-|b|}\otimes D_1$;

$with\ |b| = \sum_{i=1}^{4} b_i = 4{,}5{,}6\ and\ b_1 \geq 1$

• $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$;

$with\ |b| = \sum_{i=1}^{4} b_i = 5{,}6{,}7\ and\ b_1 \geq 2$

• $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$;

$with|b| = b_1 + b_2 + b_3 = 4{,}5{,}6{,}7and\ b_1 \geq 2$

◦ In dimension seven (M_7) we have the sum of the following terms: • $Z_{32}yZ_{32}yZ_{21}^{(3)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{13}\otimes D_0\otimes D_1$

• $Z_{32}^{(c_1)}yZ_{32}^{(c_2)}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$;

$with\ c_1 + c_2 = 3\ and\ |b| = \sum_{i=1}^{5} b_i = 8\ and\ b_1 \geq 4$

• $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{6+|b|}\otimes D_{8-|b|}\otimes D_0$;

$with\ \ |b| = b_1 + b_2 + b_3 = 7{,}8\ and\ b_1 \geq 4$

• $Z_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{7+|b|}\otimes D_{6-|b|}\otimes D_1$;

$with\ |b| = \sum_{i=1}^{5} b_i = 5{,}6\ and\ b_1 \geq 1$

• $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$;

$with\ |b| = \sum_{i=1}^{5} b_i = 6{,}7\ and\ b_1 \geq 2$

• $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$;

$with\ |b| = \sum_{i=1}^{4} b_i = 5{,}6{,}7\ and\ b_1 \geq 2$

◦ In dimension eight (M_8) we have the sum of the following terms:

• $Z_{32}yZ_{32}yZ_{32}yZ_{21}^{(4)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{14}\otimes D_0\otimes D_0$

• $Z_{32}yZ_{31}zZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{13}\otimes D_0\otimes D_1$

• $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(2)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{14}\otimes D_0\otimes D_0$

• $Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(b_1)}xZ_{21}^{(b_2)}xZ_{21}^{(b_3)}xZ_{21}^{(b_4)}xZ_{21}^{(b_5)}xD_{7+|b|}\otimes D_{7-|b|}\otimes D_0$;

$with\ \ |b| = \sum_{i=1}^{5} b_i = 6{,}7\ and\ b_1 \geq 2$

Finally In dimension nine (M_9) we have

•$Z_{32}yZ_{32}yZ_{31}zZ_{21}^{(2)}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xZ_{21}xD_{14}\otimes D_0\otimes D_0$

In [2] , it is necessary to introduce a quotient of bar complex modulo the Capelli identities relations ; the proof these relation are compatible with the boundary map $\partial_x + \partial_y + \partial_z$ is complicated [2] .

III. LASCOUX RESOLUTION OF THE PARTITION (6,5,3)

The Lascoux resolution of the weyl module associated to the partition (6,5,3) looks like this

$$0 \to D_8F\otimes D_5F\otimes D_1F \to \begin{matrix} D_8F\otimes D_4F\otimes D_2F \\ \oplus \\ D_7F\otimes D_6F\otimes D_1F \end{matrix} \to \begin{matrix} D_7F\otimes D_4F\otimes D_3F \\ \oplus \\ D_6F\otimes D_6F\otimes D_2F \end{matrix} \to D_6F\otimes D_5F\otimes D_3F \to 0$$

Where the position of the terms of the complex determined by the length of the permutations to which they correspond .The correspondence between the terms of the resolution above and permutations is as follows:

$D_6F \otimes D_5F \otimes D_3F \leftrightarrow identity$

$D_4F \otimes D_7F \otimes D_3F \leftrightarrow (12)$

$D_6F \otimes D_2F \otimes D_6F \leftrightarrow (23)$

$D_4F \otimes D_2F \otimes D_8F \leftrightarrow (123)$

$D_1F \otimes D_5F \otimes D_8F \leftrightarrow (13)$

$D_1F \otimes D_6F \otimes D_7F \leftrightarrow (132)$

Now ,the terms can be presented as below ,following Buchsbaum method [1] .

$M_0 = A_0$

$M_1 = A_1 + B_1$

$M_2 = A_2 + B_2$

$M_3 = A_3 + B_3$

$M_j = B_j$; for j=4,5,6,7,8,9.

Where A_s are the sums of the Lascoux terms and the B_s are the sums of the others.

Then the map can be defined as: $\sigma_1 : B_1 \to A_1$

If we define this map as follows:

- $Z_{21}^{(2)} x(v) \longmapsto \frac{1}{2} Z_{21} x \partial_{21}(v)$; $where\ v \in D_8 \otimes D_3 \otimes D_3$
- $Z_{21}^{(3)} x(v) \longmapsto \frac{1}{3} Z_{21} x \partial_{21}^{(2)}(v)$; $where\ v \in D_9 \otimes D_2 \otimes D_3$
- $Z_{21}^{(4)} x(v) \longmapsto \frac{1}{4} Z_{21} x \partial_{21}^{(3)}(v)$; $where\ v \in D_{10} \otimes D_1 \otimes D_3$
- $Z_{21}^{(5)} x(v) \longmapsto \frac{1}{5} Z_{21} x \partial_{21}^{(4)}(v)$; $where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{32}^{(3)} y(v) \longmapsto \frac{1}{3} Z_{32} y \partial_{32}^{(2)}(v)$; $where\ v \in D_6 \otimes D_8 \otimes D_0$

We should point out that the map σ_1 satisfies the identity :

$$\delta_{A_1A_0}\sigma_1 = \delta_{B_1B_0} \tag{3.1}$$

$$\begin{tikzcd} A_1 \arrow[rr, "\delta_{A_1A_0}"] & & A_0 = B_0 \\ & B_1 \arrow[ul, "\sigma_1"] \arrow[ur] & \end{tikzcd}$$

Where by $\delta_{A_1A_0}$ we mean the component of the boundary of the fat complex which conveys A_1 to A_0. We will use notation $\delta_{A_{i+1}A_i}$, $\delta_{A_{i+1}B_i}$ etc .Then we can define $\partial_1 : A_1 \to A_0$ as $\partial_1 = \delta_{A_1A_0}$ .It is easy to show that ∂_1 which we defined above satisfies (3.1) ,for example :

$$(\delta_{A_1A_0} \circ \sigma_1)(Z_{21}^{(2)} x(v)) = \delta_{A_1A_0}(\tfrac{1}{2} Z_{21} x \partial_{21}(v)) = \tfrac{1}{2}(\partial_{21}\partial_{21}(v)) = \partial_{21}^{(2)}(v) = \delta_{B_1B_0}(Z_{21}^{(2)} x(v))$$

At this point we are in position to define $\partial_2 : A_2 \to A_1$ as $\partial_2 = \delta_{A_2A_1} + \sigma_1 \delta_{A_2B_1}$

Proposition(3.1)

The composition $\partial_1 \circ \partial_2$ =0

Proof: [1] ,[3]

$$\partial_1 \circ \partial_2(a) = \delta_{A_1A_0} \circ (\delta_{A_2A_1}(a) + \sigma_1 \circ \delta_{A_2B_1}(a))$$

$$= \delta_{A_1A_0} \circ \delta_{A_2A_1}(a) + \delta_{A_1A_0} \circ \sigma_1 \circ \delta_{A_2B_1}(a)$$

$but\ \delta_{A_1A_0} \circ \sigma_1 = \delta_{B_1B_0}$ we have

$\partial_1 \circ \partial_2(a) = \delta_{A_1A_0} \circ \delta_{A_2A_1}(a) + \delta_{B_1B_0} \circ \delta_{A_2B_1}(a)$

Which equal to zero because the properties of the boundary map δ [1] ,so we get that $\partial_1\partial_2 = 0$.

Now we define map $\sigma_2: B_2 \to A_2$ such that

$$\delta_{A_2A_1} + \sigma_1 \circ \delta_{B_2B_1} = (\delta_{A_2A_1} + \sigma_1 \circ \delta_{A_2B_1}) \circ \sigma_2 \qquad (3.2)$$

We define this maps as follows:

- $Z_{21}xZ_{21}x(v) \mapsto 0$ $;where\ v \in D_8 \otimes D_3 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}x \mapsto 0$ $;where\ v \in D_9 \otimes D_2 \otimes D_3$
- $Z_{21}xZ_{21}^{(2)}x(v) \mapsto 0$ $;where\ v \in D_9 \otimes D_2 \otimes D_3$
- $Z_{21}^{(3)}xZ_{21}x(v) \mapsto 0$ $;where\ v \in D_{10} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}^{(2)}x(v) \mapsto 0$ $;where\ v \in D_{10} \otimes D_1 \otimes D_3$
- $Z_{21}xZ_{21}^{(3)}x(v) \mapsto 0$ $;where\ v \in D_{10} \otimes D_1 \otimes D_3$
- $Z_{21}^{(4)}xZ_{21}x(v) \mapsto 0$ $;where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}xZ_{21}^{(2)}x(v) \mapsto 0$ $;where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}^{(3)}x(v) \mapsto 0$ $;where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{21}xZ_{21}^{(4)}x(v) \mapsto 0$ $;where\ v \in D_{11} \otimes D_0 \otimes D_3$

- $Z_{32}yZ_{21}^{(3)}x \mapsto \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}(v)$ $;where\ v \in D_9 \otimes D_3 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}x(v) \mapsto \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}(v)$ $;where\ v \in D_{10} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}x(v) \mapsto \frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}(v)$ $;where\ v \in D_{11} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(6)}x(v) \mapsto \frac{1}{15}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}(v)$ $;where\ v \in D_{12} \otimes D_0 \otimes D_2$

- $Z_{32}^{(2)} y Z_{21}^{(3)} x(v) \mapsto \frac{1}{2} Z_{32} y Z_{21}^{(2)} x \partial_{31}(v) + \frac{1}{6} Z_{32} y Z_{21}^{(2)} x \partial_{21} \partial_{32}(v)$; *where* $v \in D_9 \otimes D_4 \otimes D_1$
- $Z_{32}^{(2)} y Z_{12}^{(4)} x(v) \mapsto \frac{1}{6} Z_{32} y Z_{21}^{(2)} x \partial_{21} \partial_{31}(v) + \frac{1}{12} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(2)} \partial_{32}(v)$; *where* $v \in D_{10} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(5)} x(v) \mapsto \frac{1}{30} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(2)} \partial_{31}(v) - \frac{1}{5} Z_{32} y Z_{31} z \partial_{21}^{(4)}(v)$; *where* $v \in D_{11} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(6)} x(v) \mapsto -\frac{1}{4} Z_{32} y Z_{31} z \partial_{21}^{(5)}(v) - \frac{1}{60} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(4)} \partial_{32}(v)$; *where* $v \in D_{12} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(7)} x(v) \mapsto -\frac{1}{5} Z_{32} y Z_{31} z \partial_{21}^{(6)}(v)$; *where* $v \in D_{13} \otimes D_0 \otimes D_1$
- $Z_{32} y Z_{32} y(v) \mapsto 0$; *where* $v \in D_6 \otimes D_7 \otimes D_1$
- $Z_{32}^{(3)} y Z_{21}^{(4)} x(v) \mapsto \frac{1}{3} Z_{32} y Z_{21}^{(2)} x \partial_{31}^{(2)}(v) - \frac{1}{6} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(2)} \partial_{32}^{(2)}(v) - \frac{1}{3} Z_{32} y Z_{31} z \partial_{21}^{(3)} \partial_{32}(v)$; *where* $v \in D_{10} \otimes D_4 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(5)} x(v) \mapsto \frac{1}{30} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(3)} \partial_{32}^{(2)}(v) - \frac{1}{6} Z_{32} y Z_{31} z \partial_{21}^{(3)} \partial_{31}(v)$; *where* $v \in D_{11} \otimes D_3 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(6)} x(v) \mapsto \frac{1}{45} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(4)} \partial_{32}^{(2)}(v) + \frac{1}{30} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(3)} \partial_{32} \partial_{31}(v) + \frac{1}{18} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(2)} \partial_{31}^{(2)}(v)$; *where* $v \in D_{12} \otimes D_2 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(7)} x(v) \mapsto \frac{1}{30} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(3)} \partial_{31}^{(2)}(v) + \frac{1}{45} Z_{32} y Z_{21}^{(2)} x \partial_{21}^{(4)} \partial_{32} \partial_{31}(v)$; *where* $v \in D_{13} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(8)} x(v) \mapsto -\frac{1}{15} Z_{32} y Z_{31} z \partial_{21}^{(6)} \partial_{31}(v)$; *where* $v \in D_{14} \otimes D_0 \otimes D_0$
- $Z_{32} y Z_{32}^{(2)} y(v) \mapsto 0$; *where* $v \in D_6 \otimes D_8 \otimes D_3$
- $Z_{32}^{(2)} y Z_{32} y(v) \mapsto 0$; *where* $v \in D_6 \otimes D_8 \otimes D_3$
- $Z_{32}^{(2)} y Z_{31} z(v) \mapsto \frac{1}{3} Z_{32} y Z_{31} z \partial_{32}(v)$; *where* $v \in D_7 \otimes D_7 \otimes D_0$

It is easy to show that σ_2 which is defined above satisfies (3.2) ,for example we chose one of them

$(\delta_{B_2 A_1} + \sigma_1 \delta_{B_2 B_1})(Z_{32}^{(2)} y Z_{31} z(v))$; *where* $v \in D_7 \otimes D_7 \otimes D_0$

$= \sigma_1(Z_{32}^{(3)} y \partial_{21}(v)) - Z_{21} x \partial_{32}^{(3)}(v) - \sigma_1(Z_{32}^{(2)} y \partial_{31}(v))$

$= \frac{1}{3} Z_{32} y \partial_{21} \partial_{32}^{(2)}(v) + \frac{1}{3} Z_{32} y \partial_{32} \partial_{31}(v) - Z_{21} x \partial_{32}^{(3)}(v) -$
$\frac{1}{2} Z_{32} y \partial_{32} \partial_{31}(v)$

$= \frac{1}{3} Z_{32} y \partial_{21} \partial_{32}^{(2)}(v) - \frac{1}{6} Z_{32} y \partial_{32} \partial_{31}(v) - Z_{21} x \partial_{32}^{(3)}(v)$

and

$(\delta_{A_2 A_1} + \sigma_1 \delta_{A_2 B_1})(\frac{1}{3} Z_{32} y Z_{31} z \partial_{32}(v))$

$= \sigma_1 (\frac{1}{3} Z_{32}^{(2)} y \partial_{21} \partial_{32}(v)) - \frac{1}{3} Z_{21} x \partial_{32}^{(2)} \partial_{32}(v) -$
$\frac{1}{3} Z_{32} y \partial_{31} \partial_{32}(v)$

$= \frac{1}{6} Z_{32} y \partial_{21} \partial_{32} \partial_{32}(v) + \frac{1}{6} Z_{32} y \partial_{32} \partial_{31}(v) -$
$\frac{3}{3} Z_{21} x \partial_{32}^{(3)}(v) - \frac{1}{3} Z_{32} y \partial_{32} \partial_{31}(v)$

$= \frac{1}{3} Z_{32} y \partial_{21} \partial_{32}^{(2)}(v) - \frac{1}{6} Z_{32} y \partial_{32} \partial_{31}(v) - Z_{21} x \partial_{32}^{(3)}(v)$

Proposition 3 .2

we have exactness at A_i

Proof : See [1] and [3]

Now by using σ_2 we can also define $\partial_3 : A_3 \to A_2$ by $\partial_3 = \delta_{A_3 A_2} + \sigma_2 \delta_{A_3 B_2}$

Proposition 3.3

$$\partial_2 \circ \partial_3 = 0$$

Proof :the same way used in proposition (3.1)

we need the definition of a map $\sigma_3 : B_3 \to A_3$ Such that $\delta_{B_3 A_2} + \sigma_2 \delta_{B_3 B_2} = (\delta_{A_3 A_2} + \sigma_2 \delta_{A_3 B_2}) \sigma_3$ (3.3)

As follows

- $Z_{21} x Z_{21} x Z_{21} x(v) \mapsto 0$ $\quad ; where\ v \in D_9 \otimes D_2 \otimes D_3$
- $Z_{21}^{(2)} x Z_{21} x Z_{21} x(v) \mapsto 0$ $\quad ; where\ v \in D_{10} \otimes D_1 \otimes D_3$
- $Z_{21} x Z_{21}^{(2)} x Z_{21} x(v) \mapsto 0$ $\quad ; where\ v \in D_{10} \otimes D_1 \otimes D_3$
- $Z_{21} x Z_{21} x Z_{21}^{(2)} x(v) \mapsto 0$ $\quad ; where\ v \in D_{10} \otimes D_1 \otimes D_3$
- $Z_{21}^{(3)} x Z_{21} x Z_{21} x(v) \mapsto 0$ $\quad ; where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{21} x Z_{21}^{(3)} x Z_{21} x(v) \mapsto 0$ $\quad ; where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{21} x Z_{21} x Z_{21}^{(3)} x(v) \mapsto 0$ $\quad ; where\ v \in D_{11} \otimes D_0 \otimes D_3$

- $Z_{21}^{(2)}xZ_{21}^{(2)}xZ_{21}x(v) \longmapsto 0 \quad ; where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}xZ_{21}xZ_{21}^{(2)}x(v) \longmapsto 0 \quad ; where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{21}xZ_{21}^{(2)}xZ_{21}^{(2)}x(v) \longmapsto 0 \quad ; where\ v \in D_{11} \otimes D_0 \otimes D_3$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}x(v) \longmapsto 0 \quad ; where\ v \in D_9 \otimes D_3 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}x(v) \longmapsto 0 \quad ; where\ v \in D_{10} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(2)}x(v) \longmapsto 0 \quad ; where\ v \in D_{10} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}xZ_{21}x(v) \longmapsto 0 \quad ; where\ v \in D_{11} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}^{(2)}x(v) \longmapsto 0 \quad ; where\ v \in D_{11} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(3)}x(v) \longmapsto 0 \quad ; where\ v \in D_{11} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}xZ_{21}x(v) \longmapsto 0 \quad ; where\ v \in D_{12} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}xZ_{21}^{(2)}x(v) \longmapsto 0 \quad ; where\ v \in D_{12} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}xZ_{21}^{(3)}x(v) \longmapsto 0 \quad ; where\ v \in D_{12} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(2)}xZ_{21}^{(4)}x(v) \longmapsto 0 \quad ; where\ v \in D_{12} \otimes D_0 \otimes D_2$
- $Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}x(v) \longmapsto 0 \quad ; where\ v \in D_{10} \otimes D_3 \otimes D_1$
- $Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}x(v) \longmapsto \frac{1}{4}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}(v) \quad ; where\ v \in D_{11} \otimes D_2 \otimes D_1$

- $Z_{32}^{(2)} y Z_{21}^{(3)} x Z_{21}^{(2)} x(v) \mapsto \frac{1}{2} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(3)}(v)$; *where* $v \in D_{11} \otimes D_2 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(5)} x Z_{21} x(v) \mapsto \frac{1}{10} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(4)}(v)$; *where* $v \in D_{12} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(4)} x Z_{21}^{(2)} x(v) \mapsto \frac{3}{4} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(4)}(v)$; *where* $v \in D_{12} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(3)} x Z_{21}^{(3)} x(v) \mapsto Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(4)}(v)$; *where* $v \in D_{12} \otimes D_1 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(6)} x Z_{21} x(v) \mapsto -\frac{1}{60} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)}(v)$; *where* $v \in D_{13} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(5)} x Z_{21}^{(2)} x(v) \mapsto \frac{1}{5} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)}(v)$; *where* $v \in D_{13} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(4)} x Z_{21}^{(3)} x(v) \mapsto \frac{7}{6} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)}(v)$; *where* $v \in D_{13} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)} y Z_{21}^{(3)} x Z_{21}^{(4)} x(v) \mapsto \frac{7}{6} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)}(v)$; *where* $v \in D_{13} \otimes D_0 \otimes D_1$
- $Z_{32} y Z_{32} y Z_{21}^{(3)} x(v) \mapsto$
  0 ; *where* $v \in D_9 \otimes D_4 \otimes D_1$
- $Z_{32} y Z_{32} y Z_{21}^{(4)} x(v) \mapsto$
  0 ; *where* $v \in D_{10} \otimes D_3 \otimes D_1$
- $Z_{32} y Z_{32} y Z_{21}^{(5)} x(v) \mapsto -\frac{1}{10} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(3)}(v)$; *where* $v \in D_{11} \otimes D_2 \otimes D_1$
- $Z_{32} y Z_{32} y Z_{21}^{(6)} x(v) \mapsto -\frac{1}{10} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(4)}(v)$; *where* $v \in D_{12} \otimes D_1 \otimes D_1$
- $Z_{32} y Z_{32} y Z_{21}^{(7)} x(v) \mapsto -\frac{1}{15} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)}(v)$; *where* $v \in D_{13} \otimes D_0 \otimes D_1$
- $Z_{32}^{(3)} y Z_{21}^{(4)} x Z_{21} x(v) \mapsto \frac{1}{6} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(2)} \partial_{31}(v) -$
  $\frac{1}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(3)} \partial_{32}(v)$
  ; *where* $v \in D_{11} \otimes D_3 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(5)} x Z_{21} x(v) \mapsto -\frac{1}{6} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(3)} \partial_{31}(v)$; *where* $v \in D_{12} \otimes D_2 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(4)} x Z_{21}^{(2)} x(v) \mapsto -\frac{1}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(3)} \partial_{31}(v) -$
  $\frac{2}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(4)} \partial_{32}(v)$
  ; *where* $v \in D_{12} \otimes D_2 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(6)} x Z_{21} x(v) \mapsto$
  0 ; *where* $v \in D_{13} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(5)} x Z_{21}^{(2)} x(v) \mapsto -\frac{1}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(4)} \partial_{31}(v)$; *where* $v \in D_{13} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(4)} x Z_{21}^{(3)} x(v) \mapsto -\frac{2}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(4)} \partial_{31}(v) -$
  $\frac{10}{9} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)} \partial_{32}(v)$
  ; *where* $v \in D_{13} \otimes D_1 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(7)} x Z_{21} x(v) \mapsto \frac{4}{45} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)} \partial_{31}(v)$; *where* $v \in D_{14} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(6)} x Z_{21}^{(2)} x(v) \mapsto \frac{14}{45} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)} \partial_{31}(v)$; *where* $v \in D_{14} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(5)} x Z_{21}^{(3)} x(v) \mapsto \frac{1}{15} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)} \partial_{31}(v)$; *where* $v \in D_{14} \otimes D_0 \otimes D_0$
- $Z_{32}^{(3)} y Z_{21}^{(4)} x Z_{21}^{(4)} x(v) \mapsto -\frac{1}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(5)} \partial_{31}(v)$; *where* $v \in D_{14} \otimes D_0 \otimes D_0$
- $Z_{32} y Z_{32}^{(2)} y Z_{21}^{(4)} x(v) \mapsto -\frac{1}{3} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(2)} \partial_{32}(v)$; *where* $v \in D_{10} \otimes D_4 \otimes D_0$
- $Z_{32} y Z_{32}^{(2)} y Z_{21}^{(5)} x(v) \mapsto -\frac{1}{6} Z_{32} y Z_{31} z Z_{21} x \partial_{21}^{(2)} \partial_{31}(v)$; *where* $v \in D_{11} \otimes D_3 \otimes D_0$

- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(6)}x(v) \mapsto 0$; $where\ v \in D_{12} \otimes D_2 \otimes D_0$
- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(7)}x(v) \mapsto 0$; $where\ v \in D_{13} \otimes D_1 \otimes D_0$
- $Z_{32}yZ_{32}^{(2)}yZ_{21}^{(8)}x(v) \mapsto -\frac{1}{30}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{31}(v)$; $where\ v \in D_{14} \otimes D_0 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(4)}x(v) \mapsto -\frac{1}{3}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)$; $where\ v \in D_{10} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(5)}x(v) \mapsto \frac{1}{20}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{6}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{31}(v)$; $where\ v \in D_{11} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(6)}x(v) \mapsto \frac{1}{20}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{31}(v) + \frac{1}{20}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; $where\ v \in D_{12} \otimes D_2 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(7)}x(v) \mapsto \frac{1}{30}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{32}(v) + \frac{1}{20}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v)$; $where\ v \in D_{13} \otimes D_1 \otimes D_0$
- $Z_{32}^{(2)}yZ_{32}yZ_{21}^{(8)}x(v) \mapsto 0$; $where\ v \in D_{14} \otimes D_0 \otimes D_0$
- $Z_{32}yZ_{32}yZ_{32}y(v) \mapsto 0$; $where\ v \in D_6 \otimes D_8 \otimes D_0$
- $Z_{32}yZ_{31}zZ_{21}^{(2)}x(v) \mapsto \frac{1}{2}Z_{32}yZ_{31}zZ_{21}x\partial_{21}(v)$; $where\ v \in D_9 \otimes D_4 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(3)}x(v) \mapsto \frac{1}{3}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}(v)$; $where\ v \in D_{10} \otimes D_3 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(4)}x(v) \mapsto \frac{1}{10}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}(v)$; $where\ v \in D_{11} \otimes D_2 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(5)}x(v) \mapsto 0$; $where\ v \in D_{12} \otimes D_1 \otimes D_1$
- $Z_{32}yZ_{31}zZ_{21}^{(6)}x(v) \mapsto 0$; $where\ v \in D_{13} \otimes D_0 \otimes D_1$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(2)}x(v) \mapsto \frac{1}{6}Z_{32}yZ_{31}zZ_{21}x\partial_{21}\partial_{32}(v) + \frac{1}{3}Z_{32}yZ_{31}zZ_{21}x\partial_{31}(v)$; $where\ v \in D_9 \otimes D_5 \otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(3)}x(v) \mapsto \frac{1}{6}Z_{32}yZ_{31}zZ_{21}x\partial_{21}\partial_{31}(v)$; $where\ v \in D_{10} \otimes D_4 \otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(4)}x(v) \mapsto \frac{1}{30}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{32}(v)$; $where\ v \in D_{11} \otimes D_3 \otimes D_0$
- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(5)}x(v) \mapsto \frac{1}{12}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{31}(v) + \frac{1}{60}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; $where\ v \in D_{12} \otimes D_2 \otimes D_0$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(6)}x(v) \mapsto \frac{1}{15}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(4)}\partial_{31}(v) + \frac{1}{45}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(5)}\partial_{32}(v)$

$; where\ v \in D_{13}\otimes D_1\otimes D_0$

- $Z_{32}^{(2)}yZ_{31}zZ_{21}^{(7)}x(v) \mapsto 0$ $;where\ v \in D_{14}\otimes D_0\otimes D_0$

- $Z_{32}yZ_{32}yZ_{31}z(v) \mapsto 0$ $;where\ v \in D_7\otimes D_7\otimes D_0$

Again we can show that σ_3 which defined above satisfies the condition (3.3) ,and we chose one of them as an example

- $(\delta_{B_3A_2} + \sigma_2\delta_{B_3B_2})(Z_{32}yZ_{32}yZ_{21}^{(5)}x(v))$ $;\ where\ v \in D_{11}\otimes D_2\otimes D_1$

$= \sigma_2(2Z_{32}^{(2)}yZ_{21}^{(5)}x(v)) - \sigma_2(Z_{32}yZ_{21}^{(5)}x\partial_{32}(v)) - \sigma_2(Z_{32}yZ_{21}^{(4)}x\partial_{31}(v)) + \sigma_2(Z_{32}yZ_{32}y\partial_{21}^{(5)}(v))$

$= \frac{2}{30}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{2}{5}Z_{32}yZ_{31}z\partial_{21}^{(4)}(v) - \frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v)$

$= -\frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{2}{5}Z_{32}yZ_{31}z\partial_{21}^{(4)}(v) - \frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}(v)$

and

$(\delta_{A_3A_2} + \sigma_2\delta_{A_3B_2})(-\frac{1}{10}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}(v))$

$= \sigma_2(\frac{1}{10}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(3)}(v)) - \frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}^{(3)}(v) + \sigma_2(\frac{1}{10}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(3)}(v)) -$

$\frac{4}{10}Z_{32}yZ_{31}z\partial_{21}^{(4)}(v)$

$= -\frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{10}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{2}{5}Z_{32}yZ_{31}z\partial_{21}^{(4)}(v)$

So from all we have done above we have the complex

$$0 \longrightarrow A_3 \xrightarrow{\partial_3} A_2 \xrightarrow{\partial_2} A_1 \xrightarrow{\partial_1} A_0 \quad (3.4)$$

Where ∂_i defined by:

- $\partial_1(Z_{21}x(v)) = \partial_{21}(v)$
- $\partial_1(Z_{32}y(v)) = \partial_{32}(v)$
- $\partial_2(Z_{32}yZ_{21}^{(2)}x(v)) = \frac{1}{2}Z_{21}x\partial_{21}\partial_{32}(v) + Z_{21}x\partial_{31}(v) - Z_{32}y\partial_{21}^{(2)}(v)$
- $\partial_2(Z_{32}yZ_{31}z(v)) = \frac{1}{2}Z_{32}y\partial_{32}\partial_{21}(v) - Z_{21}x\partial_{32}^{(2)}(v) - Z_{32}y\partial_{32}^{(2)}(v)$

finally, we defined the map ∂_3 by :

$\partial_3(Z_{32}yZ_{31}zZ_{21}x(v)) = Z_{32}yZ_{21}^{(2)}x\partial_{32}(v) + Z_{32}yZ_{31}z\partial_{21}(v)$

proposition 4

The complex $0 \longrightarrow A_3 \xrightarrow{\partial_3} A_2 \xrightarrow{\partial_2} A_1 \xrightarrow{\partial_1} A_0 \longrightarrow K_{(6,5,3)}F$ is exact .

Proof: see [1] and [3] .

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